PORTFOLIO MANAGEMENT

4. Performance Evaluation

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Portfolio Performance Measurement

- If we are paying an active manager to provide superior performance, how can we be sure we are getting what we pay for?
- Techniques:
    - Risk-adjusted measures
    - Market-timing measures
    - Performance attribution and “style” analysis

Dollar-and Time-Weighted Returns

- Dollar-weighted returns
  - Internal rate of return considering the cash flow from or to investment
  - Returns are weighted by the amount invested in each stock
- Time-weighted returns
  - Not weighted by investment amount
  - Equal weighting

Dollar-and Time-Weighted Returns

Period	Action
0	Purchase 1 share at \$50
1	Purchase 1 share at \$53
	Stock pays a dividend of \$2 per share
2	Stock pays a dividend of \$2 per share
	Stock is sold at \$108 per share

Dollar-Weighted Returns

Period	Cash Flow
0	-50 share purchase
1	+2 dividend -53 share purchase
2	+4 dividend + 108 shares sold

Internal Rate of Return:

$$0 = -50 - \frac{51}{(1+r)^1} + \frac{112}{(1+r)^2}$$

$$r = 7.117\%$$

Dollar-and Time-Weighted Returns

- Problem: consider the following sequence of Cash Flows. Compute the Dollar-Weighted Returns

Period	**Action**
0	Purchase 1 share at \$50
1	Stock pays a dividend of \$2 per share
2	Stock pays a dividend of \$2 per share
	Stock is sold at \$54 per share

- Comment

Time-Weighted Returns

$$r_1 = \frac{53 - 50 + 2}{50} = 10\%$$
$$r_2 = \frac{54 - 53 + 2}{53} = 5.66\%$$

Simple Average Return:

$$(10\% + 5.66\%)/2 = 7.83\%$$

Rk: Time-Weighted Return (TWR) measures the performance of the portfolio manager. The amount of funds invested is 'neutralized' in the calculation of TWR because contributions and withdrawals by the client are not under the control of the fund manager. The time-weighted return over a certain period depends only on the length of this period and not on the amount invested - the return is 'time-weighted'.

Averaging Returns

- Arithmetic Mean:

$$r = \sum_{t=1}^{n} \frac{r_t}{n}$$

Example Average: $(.10 + .0566)/2 = 7.83\%$

- Geometric Mean:

$$r = \left[\prod_{t=1}^{n} (1 + r_t)\right]^{1/n} - 1$$

Example Average: $[(1.1)(1.0566)]^{1/2} - 1 = 7.81\%$

Comparison of Geometric and Arithmetic Means

- Past Performance - generally the geometric mean is preferable to arithmetic
- Predicting Future Returns- generally the arithmetic average is preferable to geometric because it is an unbiased estimator (Geometric has downward bias).

Risk Adjusted Performance Measures

- How do you discriminate between higher returns due to skillful management, and higher returns due simply to higher risk? In other words, what return should we have expected, for the same level of risk, from a manager with no particular skill?
- Did our manager do better than this?
- What's the relevant measure of risk?
- What's the relevant benchmark portfolio?

Three Widely Used Risk Adjusted Performance Measures Based on the CAPM

- Assumptions: (1) The CML and the SML are applicable to the pricing of securities. (2) Borrowing and lending takes place at the risk-free rate. (3) Construction of the CML and the SML is a function of publicly available information.
- Given the above assumptions, investors may attempt to employ private information to identify undervalued and overvalued securities. One source of legal private information is the output of unique techniques of analysis of publicly available data.

Jensen Index (also called Jensen's Alpha)

- Jensen's Index is the vertical distance from the SML. It is the excess return above or below the security market line. It can be interpreted as a measure of how much the portfolio "beat the market."
- Expected Returns views:

$$J_P \equiv \alpha_P = \underbrace{\mathbb{E}\left[R_P\right]}_{\text{Expected Return}} - \underbrace{\left(R_f + \beta_P\left(\mathbb{E}\left[R_M\right] - R_f\right)\right)}_{\text{Expected Return on the SML (CAPM)}} \quad (1)(2)$$

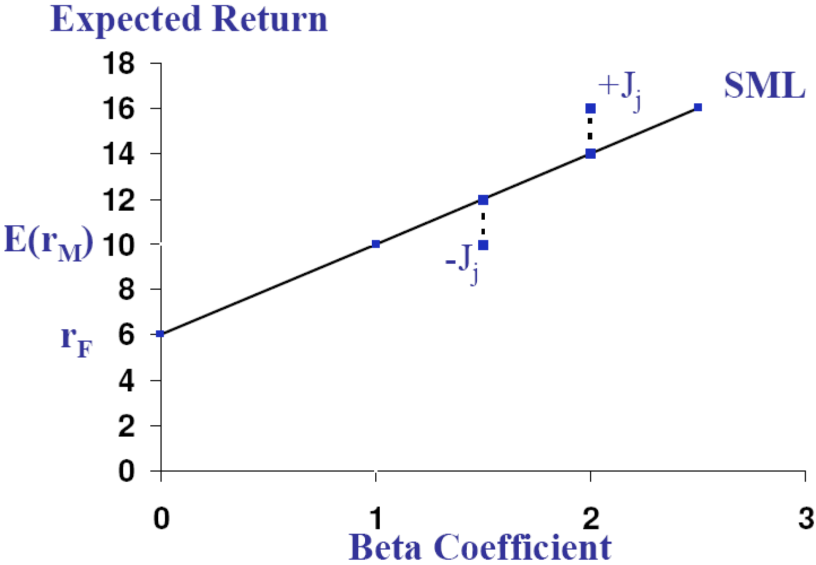
- Past Returns views:

$$\widehat{J}_P \equiv \widehat{\alpha}_P = \overline{R}_P - \left(\overline{R}_f + \widehat{\beta}_P\left(\overline{R}_M - \overline{R}_f\right)\right)$$

Jensen Index (also called Jensen's Alpha)

- The Jensen Index is sensitive only to depth and not to breadth:
  - Depth: Magnitude of excess returns.
  - Breadth: Magnitude of residual variance (e.g., Is the portfolio well diversified?)
- Note: Since beta is the risk measure:
  - Only systematic risk, and not residual variance is relevant.
  - The Jensen Index has been used for individual securities as well as portfolios. (No one expects individual securities to be well diversified).

The Jensen Index: A Graphical Illustration



Treynor Ratio

- Treynor's Ratio is the slope of a straight line going through the risk-free rate of return. The Treynor Ratio may also be defined as the risk premium earned per unit of risk taken, where beta is the risk measure.
- Evaluation of Expected Returns:

$$T = \frac{\mathbb{E}\left[R_P\right] - R_f}{\beta_P} \tag{3}$$

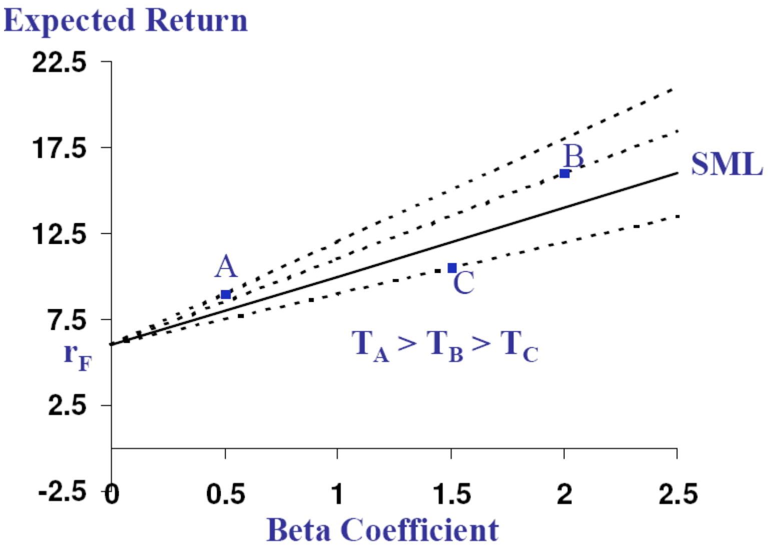
- Evaluation of Past Returns:

$$\widehat{T} = \frac{\overline{R}_P - \overline{R}_f}{\widehat{\beta}_P} \tag{4}$$

Treynor Ratio

- Similarities With the Jensen Index:
  - Since the beta coefficient is the risk measure, the Treynor Ratio (like the Jensen Index) is insensitive to breadth (i.e., it ignores residual variance).
  - Furthermore, with beta as the risk measure, the Treynor Ratio is applicable for individual securities as well as for portfolios.

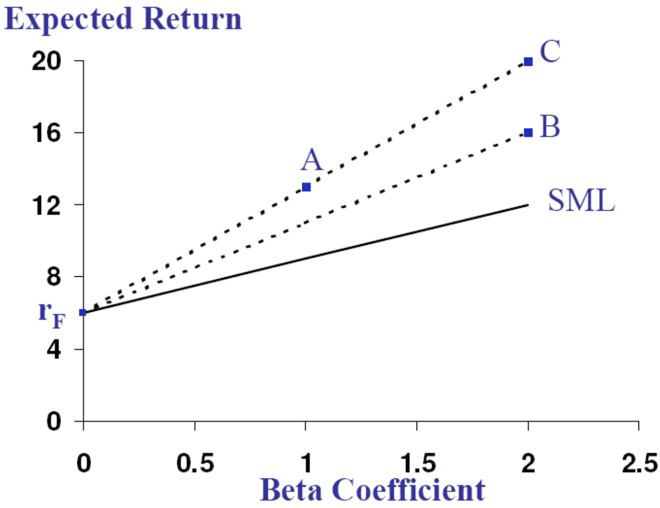
The Treynor Ratio: A Graphical Illustration



An Advantage of the Treynor Ratio Over the Jensen Index

- The Treynor Ratio is advantageous over the Jensen Index in that it takes the opportunity to lever excess returns into account when ranking alternatives.
- Example on the Following Graph:
  - An investor could borrow at the risk-free rate, and invest the proceeds in security (A) in order to obtain portfolio (C). Note that portfolio (C) dominates security (B):
  - $E(r_C) > E(r_B)$ Yet $\beta_C = \beta_B$
  - Note also that comparison between (A) and (C) gives: $J_C > J_A$ however $T_C = T_A$
- Treynor Ratio Versus Jensen Index:
  - $T_A > T_B$ However $J_A = J_B$

Treynor Ratio Versus the Jensen Index



Sharpe ratio

- The Sharpe ratio or index is a reward-to-risk ratio that focuses on total risk.
- The Sharpe Ratio is the slope of a straight line going through the risk-free rate of return. The Sharpe Ratio may also be defined as the risk premium earned per unit of risk taken, when the standard deviation of return is the risk measure.
- Evaluation of Expected Returns:

$$SR_P = \frac{\mathbb{E}\left[R_P\right] - R_f}{\sigma\left(R_P\right)} \tag{5}$$

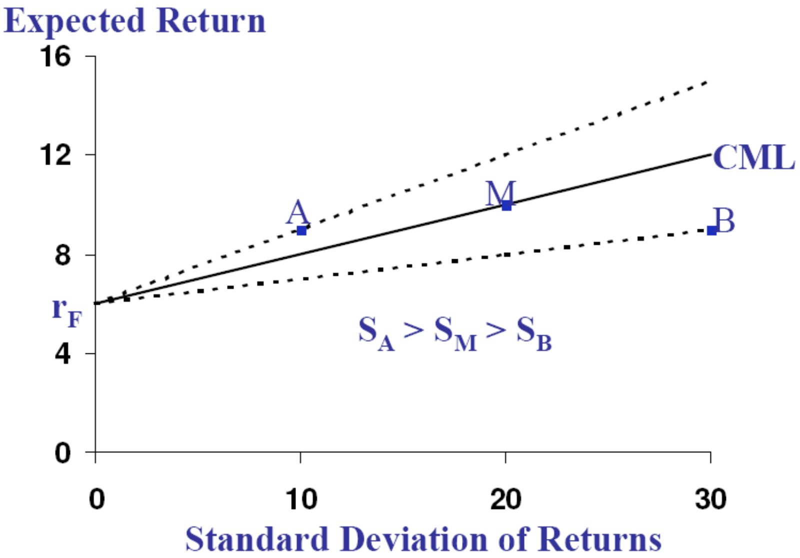
- Evaluation of Past Returns:

$$\widehat{SR}_P = \frac{\overline{R}_P - \overline{R}_f}{\sigma\left(R_P\right)} \tag{6}$$

Sharpe ratio

- The Sharpe ratio is sensitive to both:
  - Depth: Magnitude of excess returns
  - Breadth: Diversification (residual variance)
- Note: Since the standard deviation of returns is the risk measure, the Sharpe Ratio is only appropriate for portfolios and not for individual securities.

The Sharpe Ratio: A Graphical Illustration



CAPM Measures When the Market Index is Inefficient

- Note that if the market index used is inefficient, securities and portfolios plot above and below the Security Market Line. Therefore, we cannot tell if a portfolio's position relative to the SML is due to performance, or simply due to the inefficiency of the market index.
- In other words, a security's or portfolio's position relative to the SML is sensitive to the inefficient proxy chosen to represent the true market portfolio.

Comparing Performance Measures.

- Because the performance rankings can be substantially different, which performance measure should we use?

Sharpe ratio:

- Appropriate for the evaluation of an entire portfolio.
- Penalizes a portfolio for being undiversified, because in general, total risk $\approx$ systematic risk only for relatively well-diversified portfolios.

Comparing Performance Measures.

Treynor ratio and Jensen's alpha:

- Appropriate for the evaluation of securities or portfolios for possible inclusion into an existing portfolio.
- Both are similar, the only difference is that the Treynor ratio standardizes returns, including excess returns, relative to beta.
- Both require a beta estimate (and betas from different sources can differ a lot).

Performance Measure Based on the APT

- Two Factor Model Example:

$$A_P = \mathbb{E}\left[R_P\right] - \left(R_f + \beta_{1,P}\left(\mathbb{E}\left[I_1\right] - R_f\right) + \beta_{2,P}\left(\mathbb{E}\left[I_2\right] - R_f\right)\right)$$

- Note: The measure above is similar to the CAPM Jensen Index. It reflects only depth, and not breadth of performance.
- Problem: What is the appropriate factor structure?
- **Do the exercises in the file Data Alpha 1.xls**

Statistics of the Sharpe ratio

- SR must be estimated on a sample of data. It is done with error.
- How accurate is the estimation of the SR?
- Lo (2002) derived the statistical distribution of the Sharpe ratio under several sets of assumptions for the statistical behavior of the return series on which the Sharpe ratio is based.
- Lo shows that confidence intervals, standard errors, and hypothesis tests can be computed for the estimated Sharpe ratio in much the same way that they are computed for regression coefficients such as portfolio alphas and betas.
- The accuracy of Sharpe ratio estimators hinges on the statistical properties of returns, and these properties can vary considerably among portfolios, strategies, and over time. For example, Sharpe ratios are likely to be more accurately estimated for mutual funds than for hedge funds.

Statistics of the Sharpe ratio

- Simplest case: IID returns. Lo (2002) shows that the asymptotic distribution of the Sharpe ratio estimator is given by:

$$\sqrt{T}\left(\widehat{RS} - RS\right) \overset{a}{\sim} N\left(0, 1 + \frac{1}{2}RS^2\right)$$

- Table 1 reports values of standard errors (SEs) for the Sharpe ratio estimator $\widehat{RS}$ $\left(SE\left(\widehat{RS}\right) \overset{a}{=} \sqrt{\left(1 + \frac{1}{2}RS^2\right) / T}\right)$ for various combinations of Sharpe ratios and sample sizes.
- For example, for 60 observations, in a sample of 60 observations, the standard error of the Sharpe ratio estimator is 0.188 when the true Sharpe ratio is 1.50 but is 0.303 when the true Sharpe ratio is 3.00.
- This implies that the performance of investments such as hedge funds, for which high Sharpe ratios are one of the primary objectives, will tend to be less precisely estimated.

Statistics of the Sharpe ratio

	Sample Size, T					
Sharpe Ratio	12	24	36	48	60	120
0,5	0,306	0,217	0,177	0,153	0,137	0,097
0,75	0,327	0,231	0,189	0,163	0,146	0,103
1	0,354	0,250	0,204	0,177	0,158	0,112
1,25	0,385	0,272	0,222	0,193	0,172	0,122
1,5	0,421	0,298	0,243	0,210	0,188	0,133
1,75	0,459	0,325	0,265	0,230	0,205	0,145
2	0,500	0,354	0,289	0,250	0,224	0,158
2,25	0,542	0,384	0,313	0,271	0,243	0,172
2,5	0,586	0,415	0,339	0,293	0,262	0,185
2,75	0,631	0,446	0,364	0,316	0,282	0,200
3	0,677	0,479	0,391	0,339	0,303	0,214

Table: Asymptotic Standard Errors of Sharpe Ratio Estimators for Combinations of Sharpe Ratio and Sample Size

Statistics of the Sharpe ratio

- Many studies have documented various violations of the assumption of IID returns for financial securities.
- Ignoring the impact of serial correlation in hedge fund returns can yield annualized Sharpe ratios that are overstated by more than 65 percent.
- Lo (2002) derives an estimator of the Sharpe ratio in the non-IID case.

Information Ratio

- Today, the use of benchmark portfolios to evaluate the relative performance of portfolio managers is common practice in the financial management industry.
- To take account of this characteristic, the Information Ratio has been proposed. It is a type of generalized Sharpe ratio (portfolio performance is compared to a benchmark portfolio and no longer to the risk free rate)
- Thus, the relevant notion of risk becomes a relative risk measure, the tracking-error.
- Information Ratio (IR):

$$IR = \frac{R_P - R_B}{T} \tag{7}$$

- Tracking-error variance:

$$\begin{aligned} T^2 &= \sigma^2 \left(R_p - R_B \right) = \sigma_B^2 + \sigma_P^2 - 2\rho\sigma_B\sigma_P \\ &= \sigma_P^2 + \sigma_B^2 \left(1 - 2\beta_P \right) \end{aligned}$$

Market Timing

- In its pure form, market timing involves shifting funds between a market-index portfolio and a safe asset.
- If the proportion between risky asset and riskfree asset is constant, the beta of the entire portfolio remains the same over time.
- If the portfolio manager shifts funds from the riskfree assets to the risky asset in anticipation of the rise in market return, then the slope rises, the beta rises (see figures below).
- Thus, adjusting portfolio for up and down movements in the market:
  - Low Market Return - low ßeta - e.g. you anticipate a poor stock market, and move money into bonds. This results in a lower portfolio beta. When the stock market does poorly, your portfolio does not drop by as much.
  - High Market Return - high ßeta - e.g., you anticipate a better market and move money back into stocks. This results in a higher beta. When the market does improve, your portfolio rises up by more.

Market Timing

- That is, there is a regime shift in the regression analysis. To capture the regime shift, we can formulate the several regression models as:
- Treynor and Mazuy (1966):

$$r_P - r_f = a + b(r_M - r_f) + c(r_M - r_f)^2 + \varepsilon_P$$

Look for positive, significant $c > 0$.

- Furthermore, alpha, a, is always interpreted as the ability of the manager to select securities with an average return higher than that predicted by the CAPM. Thus, this model allows to know if the manager has selected stocks that have outperformed relative to the CAPM while doing some market timing. However, this method does not allow a quantification of the effect of market timing.
- Henriksson and Merton (1981):

$$r_P - r_f = a + b(r_M - r_f) + c(r_M - r_f)D + \varepsilon_P$$

$$\begin{aligned} D &= 1 \text{ if } r_m > r_f \\ &= 0 \text{ otherwise (D is a dummy variable)} \end{aligned}$$

Market Timing

Characteristic lines: Panel A: no market timing. Panel B: beta increases with expected market excess return. Panel C: market timing with only two values of beta.

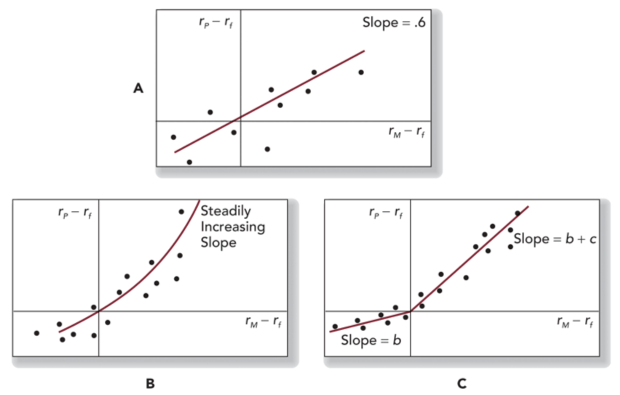


FIGURE 24.5 Characteristic lines. *Panel A:* No market timing, beta is constant. *Panel B:* Market timing, beta increases with expected market excess return. *Panel C:* Market timing with only two values of beta.

Performance Attribution

Good investment performance depends on being in the right asset (e.g. stocks vs. bonds etc) and in the right securities at the right time. Performance attribution separates over / under performance into these two components, so you can see how much asset allocation added / subtracted from your portfolio's performance, and likewise how much security picking added / subtracted.

- Decomposing overall performance into components
- Components are related to specific elements of performance
- Example components
  - Broad Allocation
  - Industry
  - Security Choice
  - Up and Down Markets

Performance Attribution

Set up a 'Benchmark' or 'Bogey' portfolio

- Use indexes for each component
- Use target weight structure
- Calculate the return on the 'Bogey' and on the managed portfolio
- Explain the difference in return based on component weights or selection
- Summarize the performance differences into appropriate categories

Performance Attribution

- Decomposing performance
- For each asset class i:
  - Benchmark weights: w_{bi}
  - Benchmark returns: R_{bi}
  - Portfolio weights:w_{pi}
  - Portfolio weights: R_{pi}

$$\text{Excess Perf} = \sum_i w_{pi} R_{pi} - \sum_i w_{bi} R_{bi}$$

$$= \underbrace{\sum_i (w_{pi} - w_{bi}) R_{bi}}_{\substack{\text{Asset allocation} \\ \text{contribution}}} + \underbrace{\sum_i w_{bi} (R_{pi} - R_{bi})}_{\substack{\text{Security selection} \\ \text{contribution}}} + \underbrace{\sum_i (w_{pi} - w_{bi})(R_{pi} - R_{bi})}_{\text{Interaction}}$$

- The term $\sum_i (w_{pi} - w_{bi}) R_{bi}$ is not the right way to caclulate the asset allocation contribution. Consider the following example:

Classe	w_{pi}	w_{bi}	R_{bi}	$(w_{pi} - w_{bi}) R_{bi}$
1	25	15	8	$0,8$
2	40	45	10	$-0,5$
3	35	40	12	$-0,6$
Total	100	100	$R_b = 10,50$	$-0,3$

Performance Attribution

- Reading the last column of previous Table leads us to believe that the decision to overweight the asset class 1 was a good decision and that the origin of the overall under-performance of the portfolio is in the decision to under-weight asset classes 2 and 3.
- However, if we look closer, we see that the manager has over-weighted a class whose return is lower than the average return of the benchmark (8% against 10.5%). This choice can not be rewarded by the mechanism of performance attribution. Consequently, this approach is not relevant.

Performance Attribution

- To assess the contribution of the asset allocation of class i on the over-performance, we must consider the excess return of the asset class compared to the average return of the benchmark:

$$AA_i = (w_{pi} - w_{bi}) \left(\overline{R}_{bi} - \overline{R}_b\right) \tag{8}$$

- The different possible situations are illustrated in Table below:

	Over-performance class i: $(R_{bi} - R_b) > 0$	Under-performance class i: $(R_{bi} - R_b) < 0$
Over weighting class i: $(w_{pi} - w_{bi}) > 0$	Good decision $(w_{pi} - w_{bi}) \,.\, (R_{bi} - R_b) > 0$	Wrong decision $(w_{pi} - w_{bi}) \,.\, (R_{bi} - R_b) < 0$
Under weighting class i: $(w_{pi} - w_{bi}) < 0$	Wrong decision $(w_{pi} - w_{bi}) \,.\, (R_{bi} - R_b) < 0$	Good decision $(w_{pi} - w_{bi}) \,.\, (R_{bi} - R_b) > 0$

- By applying this method to the previous example, we obtain:

Classe	w_{pi}	w_{bi}	R_{bi}	$(w_{pi} - w_{bi})(R_{bi} - R_b)$
1	25	15	8	$-0,25$
2	40	45	10	$0,03$
3	35	40	12	$-0,08$
Total	100	100	$R_b = 10,50$	$-0,30$

- Sub-perf compared to the benchmark is mainly due to the choice of overweighting class 1
- Furthermore, global sub-perf is the same as before (-0.3%). This arises because: $\sum_{i=1}^{n} (w_{pi} - w_{bi}) R_b = 0$ (since $\sum_{i=1}^{n} w_{pi} = \sum_{i=1}^{n} w_{bi} = 1$).

Performance Attribution: Example

- Consider the following data set:

Classes	Weights portfolio w_{pi}	Weights *benchmark* w_{bi}	Return class i : portfolio R_{pi}	Return la classe i : *benchmark* R_{bi}
Stock	40	35	$13,0$	$12,0$
Bonds	40	50	$6,75$	$7,0$
Intern. Stocks	20	15	$11,0$	$11,0$
Total	100	100		$Rb = 9,35$

- **Perform the Performance attribution of this portfolio and comment your result**

Performance Attribution: Example

	Allocation Effect	Selection Effect	Interaction Effect
Stock	$0,13$	$0,35$	$0,05$
Bonds	$0,24$	$-0,13$	$0,03$
Intern. Stocks	$0,08$	$0,00$	$0,00$
Total	$0,45$	$0,23$	$0,075$

- The benchmark and the portfolio exhibit respectively over the year a return of $9,35\%$ and $10,10\%$.
- This outperformance is composed of 0.45% from the allocation effect, 0.23% from the stock picking (security selection) and 0.075% due to the interaction of the two effects.