PORTFOLIO MANAGEMENT

3. Applied Asset Allocation

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Introduction

1. The Markowitz model and active portfolio management.
2. A Note on Estimating β
3. Using the single-index model and the CAPM in the active portfolio problem
   - 3.1 Using β to get good covariance estimates
   - 3.2 Using β to get good expected-return estimates
4. An introduction to The Black and Litterman Method.
5. Conclusions

Using the Markowitz model

- With a reasonable number of securities, the number of parameters that must be estimated is huge:

  - For a portfolio of n (100) securities we need:

σ_i	n	100
$E(r_i)$	n	100
$Cov(r_i, r_j)$	$\frac{1}{2}n(n-1)$	4950
Total	$\frac{1}{2}n(n+3)$	5150

  - About how much data will we need when we have 500 securities? 1000 Securities?

- Means and covariances are estimated with error.
- Small errors in mean or covariance estimates often lead to unreasonable weights.

Using the Markowitz model

- A remedy for both of these problems is:

 1. First, we use

 - the CAPM to determine what market believes expected returns should be

 - a single factor model to calculate asset covariances

 2. then we can combine our "views" with the CAPM-derived estimates to get portfolio weights.

- The key input we will need for both of these is the set of asset βs. So, first, we must consider the problem of estimating βs.

Estimating β

1. Let $\tilde{r}_{i,t}$, $\tilde{r}_{m,t}$ and $\tilde{r}_{f,t}$ denote historical individual security, market and risk free asset returns (respectively) over some period $t = 1, 2, ..., T$.
2. The standard way to estimate a beta is to use a *characteristic line regression (single index model)*:

$$\tilde{r}_{i,t} - \tilde{r}_{f,t} = \alpha_i + \beta_i \left(\tilde{r}_{m,t} - \tilde{r}_{f,t}\right) + \widetilde{\varepsilon}_{i,t}$$

3. To estimate this, typically we would use monthly data
4. Typically use 5 years (60 months) of data.
   - Why not use more data? (10 or 20 years)
     - Parameter instability
   - Can we use weekly, daily, or intraday data?
     - Non-synchronous prices
     - Bid-ask bounce

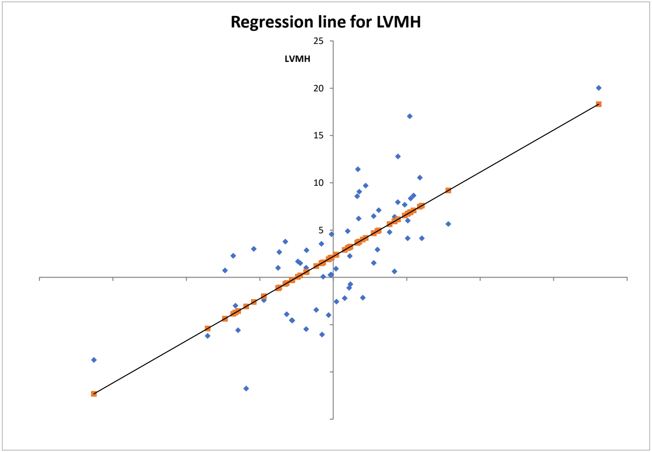
Estimating β: an example

Find below the outputs of a characteristic-line regression for LVMH stock on the Eurostoxx 50 index over the period (01:2016-12:2020, 60 observations). We consider that the riskless rate is equal to zero and thus estimate $\tilde{r}_{i,t} = \alpha_i + \beta_i . \tilde{r}_{m,t} + \tilde{\varepsilon}_{i,t}$.

Regression Statistics	
Multiple R	0.7362
R Square	0.5420
Adjusted R Square	0.5341
Standard Error	4.1158
Observations	60

	Coefficients	**standard-error**	**t-Stat**	**P-value**
Constant	2.2129	0.5321	4.1588	0.00011
EuroStoxx 50	0.8916	0.1076	8.2852	0.00000

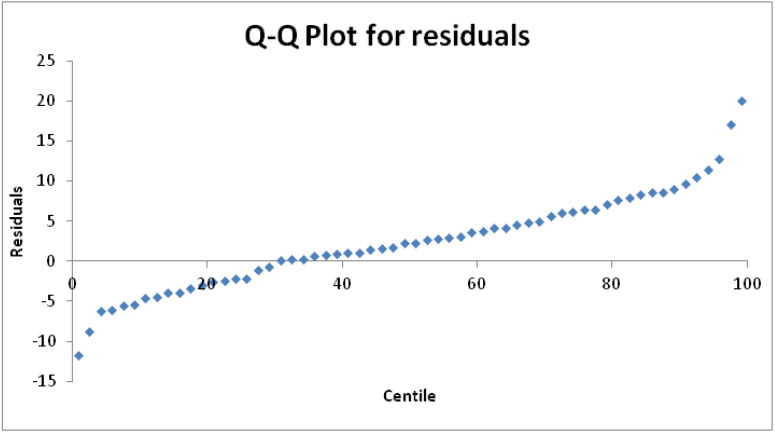
Estimating β: an example



How do we read $\widehat{\alpha}_i$, $\widehat{\beta}_i$ and $\widehat{\varepsilon}_{i,t}$ off of this scatterplot?

Estimating β: an example

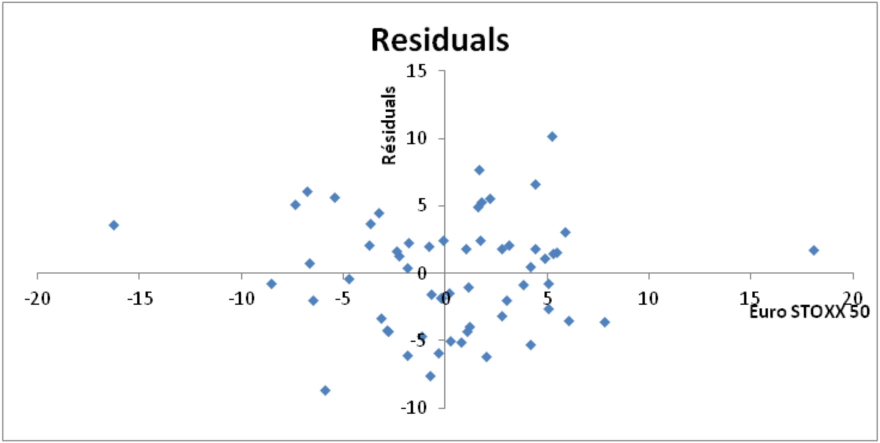
The Graph of the Q-Q Plot for the residuals of the previous regression is below



What can you infer from this graph ?

Estimating β: an example

The Graph of the residuals of the previous regression is below



What can you infer from this graph ?

Adjusted β

- Adjusted beta is a historical beta adjusted to reflect the tendency of beta to be mean-reverting.
- The motivation for adjusting beta estimates is that, on average, the beta coefficients of stocks seem to move toward 1 over time.
- When we estimate the beta over a particular sample period, we sustain some unknown sampling error of the estimated beta. The greater the difference between our beta estimate and 1, the greater is the chance that we incurred a large estimation error and that beta in a subsequent sample period will be closer to 1.
- Merrill Lynch adjusts beta estimates in a simple way. It takes the sample estimate of beta and averages with 1, using weights of two-thirds and one-third:

$$\beta_i^{Adj} \approx (1/3) .1 + (2/3) .\beta_i$$

The CAPM and Active Portfolio Management

- To create an optimal portfolio we need to estimate the minimum variance/efficient frontier, MVE portfolio, and the best capital allocation line (CAL).
- To do so we need estimates for the expected returns, variances, and covariances for all of the assets
- One way to do this is to use the sample means, variances and covariances based on past historical returns

  - we have already seen that there are problems with this approach

- An alternative approach is to "accept" the aggregate "opinion" of the market and simply hold the market portfolio

- This approach doesn't tell you what to do if you have information that you believe has not yet been incorporated into market prices.
- The approach we will discuss next incorporates "views" into the CAPM

The CAPM and Active Portfolio Management

- The approach we will instead take is to Incorporate "views" into the CAPM and Markowitz:

1. Calculate β's for the securities we plan to hold using the methods discussed earlier .

2. Using these β's, calculate $\sigma_{i,j}$'s and $E(r_i)$'s, assuming the CAPM holds exactly.

3. Then, incorporate our information by carefully "perturbing" the values away from the CAPM-calculated values.

4. Finally, using these estimates and the Markowitz portfolio optimization tools (from Lecture 1), determine our optimal portfolio weights.

- Note that if

(i) We start with all of the assets in market portfolio

(ii) We use the unmodified $\sigma_{i,j}$'s and $E(r_i)$'s we get from step 2

Then the weights we calculate in step (4) will be exactly those of the market portfolio. Why??

The single index model to estimate covariances

- Instead of using sample estimates of $\sigma_{i,j}$'s for all assets, an alternative to estimate $\sigma_{i,j}$'s is to assume that a single index model describes returns:

  1. $\tilde{r}_{i,t} - \tilde{r}_{f,t} = \alpha_i + \beta_i \left(\tilde{r}_{m,t} - \tilde{r}_{f,t}\right) + \widetilde{\varepsilon}_{i,t}$, $t = 1, ..., T$

  2. for two different securities i and j, $cov(\widetilde{\varepsilon}_{i,t}, \widetilde{\varepsilon}_{j,t}) = 0$.

  - The CAPM does not imply this.
  - Is this a reasonable simplification?

- To get covariances/correlations, use:

$$\sigma_{i,j} = cov(\tilde{r}_{i,t}, \tilde{r}_{j,t}) = \beta_i \beta_j \sigma_m^2 \quad \forall i \neq j$$

and

$$\rho_{i,j} = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j}$$

- For variances:

$$\sigma_{i,i} = \sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon_i}^2$$

The single index model vs the CAPM

1. The single index model is a **statistical** model.

- It specifies that all common movements between stocks can be captured by a single index.
- It is also often called a single factor model.
- Can be generalized to multiple common factors.
- It is a statistical technique designed to estimate large covariance matrices.

2. The CAPM is an **economic** model of expected returns.

- It specifies that the market portfolio captures systematic risk.
- However, it does **allow** securities i and j to be correlated on top of what their covariance with the market portfolio implies, for instance if they are in the same industry. The only requirement is that these correlations vanish as we add more stocks.

Estimation of expected returns by the CAPM

- Start with the characteristic line regression for estimating the $\beta_i s$:

$$\tilde{r}_{i,t} - \tilde{r}_{f,t} = \alpha_i + \beta_i \left(\tilde{r}_{m,t} - \tilde{r}_{f,t}\right) + \tilde{\varepsilon}_{i,t} \tag{1}$$

- Use the CAPM, to calculate the market's beliefs about expected returns:

$$E\left(\tilde{r}_i\right) - r_f = \hat{\beta}_i \left[E\left(\tilde{r}_m\right) - r_f\right] \tag{2}$$

  - The CAPM estimate of $E\left(\tilde{r}_i\right) - r_f$ is obtained by imposing $\alpha_i = 0$
  - Note that , to get these estimates, we need an estimate for $E\left(\tilde{r}_m\right)$

- Remember, (1) is about actual realized returns, while (2) is about expected returns

  - Hence, given our estimates $\hat{\alpha}_i$, $\hat{\beta}_i$ and $\left(\overline{r}_{m,t} - \overline{r}_{f,t}\right)$, if we calculate

$$\hat{\alpha}_i + \hat{\beta}_i \left(\overline{r}_{m,t} - \overline{r}_{f,t}\right)$$

it would be the same as estimating $\left(\overline{r}_{i,t} - \overline{r}_{f,t}\right)$ using historical averages.

The CAPM and Active Portfolio Management

Then, to construct the efficient frontier based on the Single Index Model (for 100 securities) we need estimates of the following:

r_f	1	1
$E(r_m)$	1	1
σ_m^2	1	1
α_i	n	100
β_i	n	100
$\sigma_{\varepsilon_i}^2$	n	100
Total	$3n+3$	303

- This is considerably smaller than the 5150 we had before.
- Where are the $E(\widetilde{r}_i)$?
- Where are the $\sigma_{i,j} = cov(\tilde{r}_{i,t}, \tilde{r}_{j,t})$?
- What will the efficient portfolio weights be if we assume all α_i are zero? (and include all assets in our analysis)

Example

Annualized Statistics for 5 stocks form the Eurostoxx 50 (sample period: 02/28/2011-12/31/2020):

	LVMH	Bayer	Eni	Anheuser	ING	Stoxx 50	r_f
mean	20.70%	5.61%	1.77%	8.72%	7.97%	3.29%	0%
std	22.33%	27.00%	24.23%	25.27%	35.12%	16.91%	0%
α_i	17,65%	1,90%	-1,76%	5,84%	2,40%		
β_i	0.93	1.13	1.07	0.88	1.69		
σ_{ε_i}	15,92%	19,09%	16,11%	20,48%	20,39%		
R^2	49.2%	50.0%	55.8%	34.3%	66.3%		

Example (cont)

- For expected returns, let's not impose the CAPM just yet.
- To get the correlation structure, we use:

$$\rho_{i,j} = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j} \ \forall i \neq j$$

	LVMH	Bayer	Eni	Anheuser	ING
LVMH	1	0.496	0.524	0.411	0.571
Bayer	0.496	1	0.528	0.414	0.576
Eni	0.524	0.528	1	0.438	0.608
Anheuser	0.411	0.414	0.438	1	0.477
ING	0.571	0.576	0.608	0.477	1

Example (cont)

- Plugging the (1) “expected” returns, (2) return standard deviations, and (3) correlation matrix into the Excel spreadsheet we get the following weights for the tangency portfolio:

Stocks	weights
LVMH	169.04%
Bayer	-13.07%
Eni	-57.18%
Anheuser	21.43%
ING	-20.21%

- It seems unreasonable that we should hold such extreme portfolio positions.
- The equilibrium arguments that we used in developing the CAPM suggest that the market knows something about future expected returns that we don’t

Example (cont)

- Let's use the CAPM as a way of getting around this problem.
- This is equivalent to setting $\alpha_i = 0$ for all securities, or using the regression equation:

$$E\left(\tilde{r}_i\right) = r_f + \beta_i \left[E\left(\tilde{r}_m\right) - r_f\right]$$

and the past (average) return on the market to get "equilibrium" estimates of the expected returns.

Stocks	CAPM $E\left(\tilde{r}_i\right)$
LVMH	3.05%
Bayer	3.72%
Eni	3.52%
Anheuser	2.88%
ING	5.57%

Example (cont)

- With this "equilibrium" set of expected returns, we now get the portfolio weights:

Stocks	weights
LVMH	21.47%
Bayer	18.18%
Eni	24.22%
Anheuser	12.25%
ING	23.88%

- Why are the weights different?

- Why are these not the market weights? When would these be the actual market weights?

- Is this the portfolio you would want to hold, given that you were constrained to hold these five assets?

Example (cont)

- However, there may be times when we think that the market is a little wrong along one or more dimensions (a very dangerous assumption!)
- How can we combine our views with what the market expects?
- For example, suppose that:

1. We think that the "market" has underestimated the earnings that LVMH will announce next month, and that LVMH's expected return is 1% higher than the market expects.
2. Also, we have no information on the other four securities that would lead us to think that they are mispriced,
3. We believe that the historical betas, and residual standard deviations are all good estimates of their future values.

Example (cont)

- Changing the expected returns for LVMH by $+1\%$, keeping all of the other inputs the same, the new optimal portfolio weights are:

Stocks	CAPM $E(\tilde{r}_i)$	weights
LVMH	**4.05%**	53.63%
Bayer	3.72%	10.73%
Eni	3.52%	14.30%
Anheuser	2.88%	7.23%
ING	5.57%	14.10%

compared to the old allocation

Stocks	CAPM $E(\tilde{r}_i)$	weights
LVMH	**3.05%**	21.47%
Bayer	3.72%	18.18%
Eni	3.52%	24.22%
Anheuser	2.88%	12.25%
ING	5.57%	23.88%

Example (cont)

- Alternatively, suppose that we think the risk (β) of Anheuser is going to increase:

  1. We guess that Anheuser's β will rise from 0.88 to 1.
  2. We also expect that Anheuser's idiosyncratic risk σ_ε will not change.

- First, we should recalculate almost everything using the equations:

$$E\left(\tilde{r}_i\right) = r_f + \beta_i \left[E\left(\tilde{r}_m\right) - r_f\right]$$

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon_i}^2$$

$$\rho_{i,j} = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j} \ \forall i \neq j$$

Example (cont)

- The new correlations are:

	LVMH	Bayer	Eni	Anheuser	ING
LVMH	1	0.496	0.524	**0.470**	0.571
Bayer	0.496	1	0.528	**0.473**	0.576
Eni	0.524	0.528	1	**0.500**	0.608
Anheuser	**0.470**	**0.473**	**0.500**	1	**0.545**
ING	0.571	0.576	0.608	**0.545**	1

- compared to the old correlation matrix:

	LVMH	Bayer	Eni	Anheuser	ING
LVMH	1	0.496	0.524	**0.411**	0.571
Bayer	0.496	1	0.528	**0.414**	0.576
Eni	0.524	0.528	1	**0.438**	0.608
Anheuser	**0.411**	**0.414**	**0.438**	1	**0.477**
ING	0.571	0.576	0.608	**0.477**	1

- The standard deviation of Anheuser has also changed from 25.27% to 26.56%.

Example (cont)

- If, we believe that the new correlations are the real ones, but that the market still believes that the old correlations represent the future (and will not discover this information over the management period) then we would use the old expected returns, giving new portfolio weights of:

	Old		New
Stocks	CAPM $E(\tilde{r}_i)$	weights	weights
LVMH	3.05%	21.47%	24.62%
Bayer	3.72%	18.18%	20.84%
Eni	3.52%	24.22%	27.77%
Anheuser	2.88%	12.25%	-0.62%
ING	5.57%	23.88%	27.38%

Example (cont)

- If on the other hand, we believe that market already knows that the β of Anheuser is higher, and that the expected return on Anheuser is now higher to appropriately compensate for the increased systematic risk, the optimal portfolio becomes::

	Old		New	
Stocks	$E(\tilde{r}_i)$	weights	$E(\tilde{r}_i)$	weights
LVMH	3.05%	21.47%	3.05%	21.82%
Bayer	3.72%	18.18%	3.72%	18.48%
Eni	3.52%	24.22%	3.52%	24.62%
Anheuser	2.88%	12.25%	3.29%	10.82%
ING	5.57%	23.88%	5.57%	24.27%

- Alternatively, suppose that we believe that the market hasn't yet recognize that the systematic risk of Anheuser has increased, but it will soon discover it:

1. What will happen as the market finds out?
2. What should we do in this case?

Example (cont): TO DO

Analyze the impact of a change of ING beta from 1.69 to 1.4:

1. If, you believe that the new correlations are the real ones, but that the market still believes that the old correlations represent the future (and will not discover this information over the management period),
2. If on the other hand, you believe that market already knows that the β of Anheuser is higher, and that the expected return on Anheuser is now lower.

(answers in Impact Change ING beta 2021.xlsx and Markowitz Impact change ING beta.xlsx)

Forming 'views'

- Where is our information coming from?

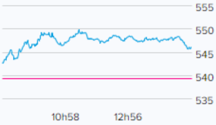
1. It may come from public sources of information which may or may not have yet been incorporated into prices.
2. It could be private information which could come from our own analysis of fundamentals.

- Nevertheless, we may not be 100% confident that our information is accurate.
- For instance, in the previous example, if we are only somewhat confident in our belief that the market will not properly adjust Anheuser's price, we might want to adjust the portfolio weights only partially.

Forming 'views'

Cours
LVMH
546.300 EUR
+1.30%

FR0000121014 MC
EURONEXT PARIS DONNÉES TEMPS RÉEL
Politique d'exécution
Cotation sur les autres places



SECTEUR
Habillement et accessoires

INDICE DE RÉFÉRENCE
CAC 40

OUVERTURE
543.200

CLÔTURE VEILLE
539.300

+ HAUT
550.000

+ BAS
542.000

LIMITE À LA BAISSE
529.900

LIMITE À LA HAUSSE
562.500

DERNIER DIVIDENDE
2 EUR

DATE DERNIER DIVIDENDE
01.12.20

CONSENSUS DES ANALYSTES AU 27/02/21*

Objectif de cours 3 mois : **564.26 EUR** - Potentiel: 3.38%

1.79

1. Acheter 2. Renforcer 3. Conserver 4. Alléger 5. Vendre

> Accès au top consensus du secteur Biens de consommation

ÉVOLUTION DES PRÉVISIONS AU 03/03/21

	RÉALISÉ 2020	ESTIM. 2021	ESTIM. 2022
Dividende net par action	6.00 EUR	6.95 EUR	7.75 EUR
Rendement	1.14%	1.32%	1.48%
Bénéfice net par action	9.32 EUR	15.24 EUR	17.65 EUR
PER	56.33	34.46	29.75

Forming 'views'

- Market price: $P_0 = 546.3$
- Our view (Fair Value: $V_0 = 564.26$): $V_0 = (1 + a)P_0 = 564.26$
  $\Rightarrow a = 3.29\%$
- If we were 100% confident in our "view," then LVMH's expected return should be higher than the CAPM return by $\alpha = 3.29\%$. It has to be added to the equilibrium return given by the CAPM because it is a current undervaluation and is thus considered as an alpha.

- This is a very large number and will likely result in extreme portfolio allocations

- Suppose that we are only "somewhat" confident, say 10%, in your "view"

- Then we might only want to use a value of $a = 10\% \times 3.29\% = 0.329\%$

- This is obviously rather ad hoc ...

Summary

- In order to use the CAPM in our mean-variance analysis, we need estimates of β. Regression analysis yields that.
- The CAPM assumes that all investors hold the same views so all of them hold the market-portfolio which is mean-variance efficient (MVE).
- The key insight of the CAPM is that what matters for expected returns is covariance risk rather than variance risk.

  - This insight remains true whether the CAPM is true or not

- We analyze how to use the CAPM (and the single index model) to build more realistic portfolios than the portfolios relying on sample estimates of the $E(r_i)$'s and $\sigma_{i,j}$'s for all assets.