PORTFOLIO MANAGEMENT

1. Asset Allocation

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Introduction

- The goal of this lecture is to understand asset-allocation theory and to provide an introduction to practice .
- Modern portfolio theory has its roots in mean-variance portfolio analysis.
  - Developed by Harry Markowitz in the early 1950's.
  - His work was the start of what has been named modern finance.
- The asset allocation problem answers the question:
  How much of your wealth should you invest in each security?

Mean-Variance Analysis

- Up until the mean-variance analysis of Markowitz became known, an investment advisor would have given you advice like:
  - If you are young you should be putting money into a couple of good growth stocks, maybe even into a few small stocks. Now is the time to take risks.
  - If you're close to retirement, you should be putting all of your money into bonds and safe stocks, and nothing into the risky stocks – don't take risks with your portfolio at this stage in your life.
- Though this advice was intuitively compelling, it was, of course, very wrong!

Mean-Variance Analysis

- We now know that the optimal portfolio of risky assets is exactly the same for everyone, **no matter what their tolerance** for risk.
    1. Investors should control the risk of their portfolio not by reallocating among risky assets, but through the split between risky and risk-free assets.
    2. The portfolio of risky assets should contain a large number of assets – it should be a **well diversified portfolio**.
- **Note**: These results are derived under the assumptions that:

    a) Either,

    - all returns are normally distributed
    - investors care only about mean return and variance.

    b) All assets are tradable.

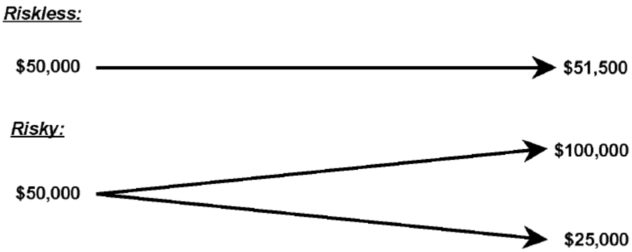
    c) There are no transaction costs.
- We will discuss the implications of relaxing these assumptions.

Mean-Variance Analysis

- In this lecture, we will decompose the analysis of this problem into two parts:
  1. What portfolio of risky assets should we hold?
  2. How should we distribute our wealth between this optimal risky portfolio and the risk-free asset?
- We will look at each problem in isolation and then bring the pieces together.
- But first, we need a theoretical framework for understanding the tradeoff between risk and return.

Framework

- An investor has the choice of investing \$50,000 in a risk-free investment or a risky investment.
  - The risky investment will either halve or double, with equal probability
  - The riskless investment will yield a certain return of \$51,500.



- How should he decide which of these investments to take?

Framework

1. Calculate the expected return for each investment

- The (simple) return on the risk free investment is:

$$r_f = \frac{51,500}{50,000} - 1 = 3\%$$

- The expected return on the risky investment is:

$$E\left(\tilde{r}\right) = \frac{1}{2}.\underbrace{\left(\frac{100}{50} - 1\right)}_{100\%} + \frac{1}{2}.\underbrace{\left(\frac{25}{50} - 1\right)}_{-50\%} = 25\%$$

2. Calculate the risk premium on the risky investment.

- **Definition**: The excess return is the return net of the risk-free rate
$$\tilde{r}^e = \tilde{r} - r_f$$
- **Definition:** The risk premium is the expected excess return
$$E\left(\tilde{r}^e\right) = E\left(\tilde{r} - r_f\right) = 1/2.\left(97\%\right) + 1/2.\left(-53\%\right) = 22\%$$

3. Calculate the riskiness of the investments

- To answer this we need a measure of risk. The measure we will use is the return variance or standard deviation;
  - For the risk-free asset, the variance is zero.
  - For the risky investment the return variance is:

$$\sigma^2(\tilde{r}) = \frac{1}{2}\left[(1-0.25)^2 + (-0.5-0.25)^2\right] = 0.56$$

and the return standard-deviation is the (positive) square-root of σ^2:

$$\sigma(\tilde{r}) = \sqrt{0.56} = 0.75 = 75\%$$

- If asset returns are normally distributed, this is a perfect measure of risk (why?).
- If returns are not normal (as is the case here), you need other assumptions to make variance a perfect proxy for riskiness.

Framework

4. Finally, we need to determine if this is a reasonable amount of risk for the extra expected return.

- We need to quantify our attitudes or preferences over risk and return.
- For a starting point, we assume people

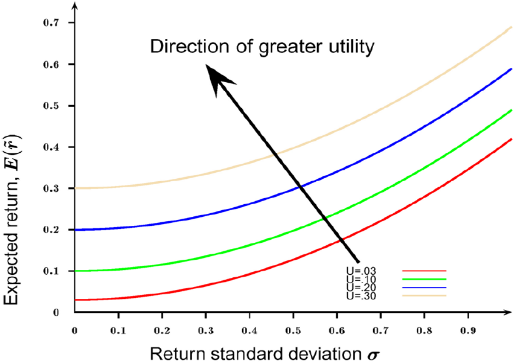
  4.1. like high expected returns $E(\tilde{r})$
  4.2. dislike high variance $\sigma^2(\tilde{r})$

- that is, investors are risk averse.
- Their utility or happiness from a pattern of returns $\widetilde{r}$ is:

$$U(\tilde{r}) = E(\tilde{r}) - \frac{1}{2}\gamma\sigma^2(\tilde{r})$$

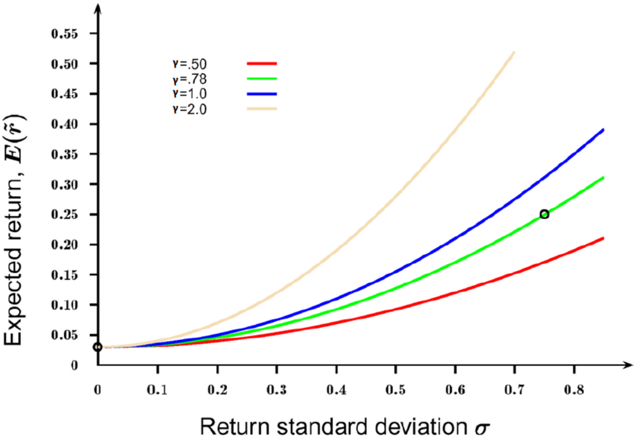
  - γ measures the investor's level of risk-aversion
  - The higher γ, the higher an investor's dislike of risk

Indifference Curves



- Investor preferences can be depicted as indifference curves.
- Each curve represents different utility levels for fixed risk aversion γ.
- Each curve traces out the combinations of $(E(\tilde{r}), \sigma(\tilde{r}))$ yielding the same level of utility U

Indifference Curves



- Each curve plots the same utility level for different risk aversion, γ.
- Higher γ implies that for a given $\sigma\left(\tilde{r}\right)$, investors require higher mean return to achieve same level of utility.

Choosing a Portfolio of Risky and Risk-free Assets

- Next we will determine how to build an optimal portfolio of risky and risk-free assets
- **Two-Fund separation** is a key result in Modern Portfolio Theory. It implies that the investment problem can be decomposed into two steps:

  1. Find the optimal portfolio of risky securities
  2. Find the best combination of the risk-free asset and this optimal risky portfolio

- We'll consider part (2) first!
- Afterwards, we will show that, if we have many risky assets, there is an optimal portfolio of these risky assets that all investors prefer.

Choosing a Portfolio of Risky and Risk-free Assets

- Calculating the return on a portfolio P consisting of one risky asset A (which will be identified as the optimal risky portfolio in the following) and a risk-free asset.
- From our example, we have

$$
\begin{aligned}
&\tilde{r}_A = \text{return on (risky) asset } A \\
&E\left(\tilde{r}_A\right) = \text{expected risky rate of return} = 25\% \\
&\sigma\left(\tilde{r}_A\right) \equiv \sigma_A = \text{standard deviation} = 75\% \\
&r_f = \text{risk-free rate} = 3\% \\
&w = \text{fraction of portfolio } P \text{ invested in risky asset } A = \text{??}
\end{aligned}
$$

Choosing a Portfolio of Risky and Risk-free Assets

- The return and expected return on a portfolio with weight w on the risky security and $1 - w$ on the risk-free asset is:

$$\begin{aligned} \tilde{r}_p &= w.\tilde{r}_A + (1 - w).r_f \\ \tilde{r}_p &= r_f + w.(\tilde{r}_A - r_f) \end{aligned}$$

$$E(\tilde{r}_p) = r_f + w.E(\tilde{r}_A^e) \quad (1)$$

- The risk (variance) of this combined portfolio is:

$$\begin{aligned} \sigma_P^2 &= E\left[(\tilde{r}_p - E(\tilde{r}_p))^2\right] \\ &= E\left[(w.\tilde{r}_A^e - w.E(\tilde{r}_A^e))^2\right] \\ &= w^2.E\left[(\tilde{r}_A - E(\tilde{r}_A))^2\right] \\ &= w^2.\sigma_A^2 \quad (2) \end{aligned}$$

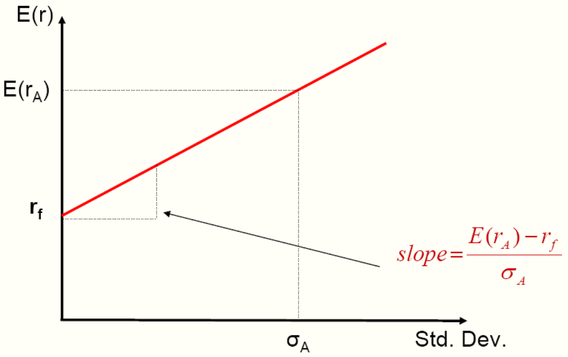
Capital Allocation Line

- We can derive the Capital Allocation Line, i.e. the set of investment possibilities created by all combinations of the risky and riskless asset.
- Combining (1) and (2), we can characterize the expected return on a portfolio with σ_P:

$$E\left(\tilde{r}_p\right) = r_f + \underbrace{\left[\frac{E\left(\tilde{r}_A\right) - r_f}{\sigma_A}\right]}_{price\ of\ risk} \underbrace{\sigma_P}_{amount\ of\ risk}$$

- The price of risk is the return premium per unit of portfolio risk (standard deviation) and depends **only** on the returns of available securities.
- The standard term for this ratio is the Sharpe Ratio.

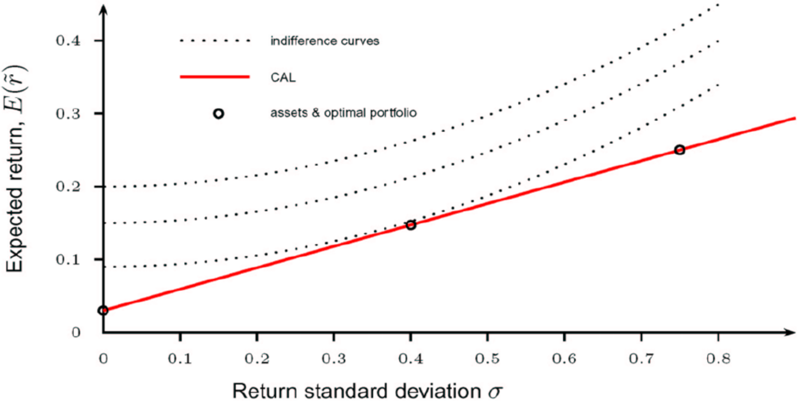
Capital Allocation Line



- The CAL shows all risk-return combinations possible from a portfolio of one risky-asset and the risk-free return.
- The slope of the CAL is the Sharpe Ratio.

Which Portfolio on the CAL?

- Which risk-return combination along the CAL do we want?
- To answer this we need the utility function!



Which Portfolio on the CAL?

- Mathematically, the optimal portfolio is the solution to the following problem:

$$U^* = \max_w U\left(\tilde{r}_p\right) = \max_w E\left(\tilde{r}_p\right) - \frac{1}{2}\gamma\sigma_P^2$$

with

$$E\left(\tilde{r}_p\right) = r_f + wE\left(\tilde{r}_A - r_f\right) \ \ and \ \sigma_P^2 = w^2\sigma_A^2$$

Combining these two equations we get:

$$\max_w U\left(\tilde{r}_p\right) = \max_w \left(r_f + wE\left(\tilde{r}_A - r_f\right) - \frac{1}{2}\gamma w^2\sigma_A^2\right)$$

Solution

$$\frac{dU}{dw}\Big|_{w=w^*} = 0 \Rightarrow w^* = \frac{E\left(\tilde{r}_A - r_f\right)}{\gamma\sigma_A^2}$$

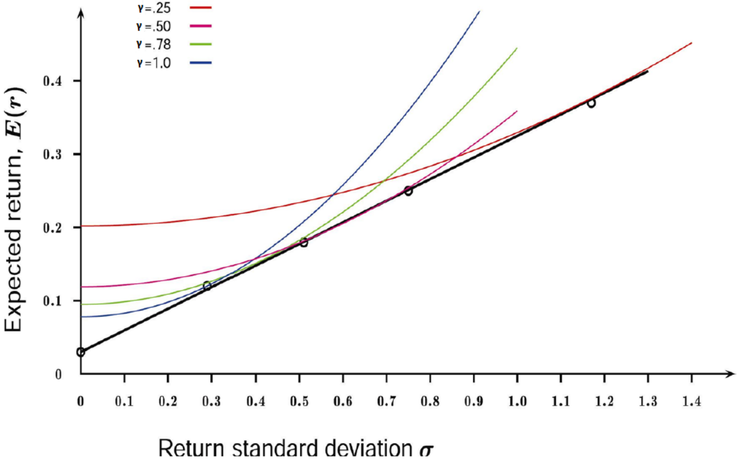
Which Portfolio on the CAL?

- For the example we are considering:

γ	w^*	$E(\tilde{r}_p)$	σ_P
0.25	1.56	0.37	1.17
0.5	0.78	0.2	0.51
0.78	0.49	0.14	0.37
1	0.39	0.12	0.29

- What is the meaning of 1.56 in the above table?
- Can you ever get a negative w ?
- How do changes in γ affect the optimal portfolio?
- How do changes in the Sharpe ratio affect the optimal portfolio?

Which Portfolio on the CAL?



- Different level of risk aversion leads to different choices.

Two Risky Assets and no Risk-Free Asset

- Now that we understand how to allocate capital between the risky and riskfree asset, we need to show that it is really true that there is a single optimal risky portfolio.
- To start, we'll ask the question:

How should you combine two risky securities in your portfolio?

- We will plot out possible set of expected returns and standard deviations for different combinations of the assets.
- **Definition**: Minimum Variance Frontier, is the set of portfolios with the lowest variance for a given expected return.

A Portfolio of Two Risky Assets

1. The expected return for the portfolio is

$$E\left(\tilde{r}_p\right) = wE\left(\tilde{r}_A\right) + \left(1 - w\right) E\left(\tilde{r}_B\right)$$

w is the fraction that is invested in asset A.

2. The variance of the portfolio is:

$$\begin{aligned} \sigma_P^2 &= E\left[\left(\tilde{r}_p - E\left(\tilde{r}_p\right)\right)^2\right] \\ &= w^2\sigma_A^2 + \left(1 - w\right)^2 \sigma_B^2 + 2w\left(1 - w\right) cov\left(\tilde{r}_A, \tilde{r}_B\right) \end{aligned}$$

or, since $\rho_{AB} = cov\left(\tilde{r}_A, \tilde{r}_B\right) / \left(\sigma_A\sigma_B\right)$

$$\sigma_P^2 = w^2\sigma_A^2 + \left(1 - w\right)^2 \sigma_B^2 + 2w\left(1 - w\right) \rho_{AB}\sigma_A\sigma_B$$

3. Notice that the variance of the portfolio depends on the correlation between the two securities.

A Portfolio of Two Risky Assets: Example

- As an example let's assume that we can trade in asset A of the previous example, and in an asset B:

Asset	$E(\tilde{r})$	σ
A	25%	75%
B	10%	25%

- To develop intuition for how correlation affects the risk of the possible portfolios, we will derive the minimum variance frontier under 3 different assumptions:

  1. $\rho_{AB} = 1$
  2. $\rho_{AB} = -1$
  3. $-1 < \rho_{AB} < +1$

Two Risky assets, $\rho_{AB} = 1$

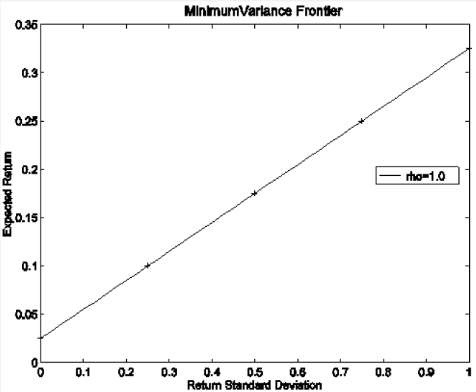
- Plug these numbers into these two equations:

$$\begin{aligned} E(\tilde{r}_P) &= 0.25.w + 0.1.(1-w) \\ &= 0.15w + 0.10 \\ \sigma_P &= 0.75w + 0.25(1-w) \\ &= 0.50w + 0.25 \end{aligned}$$

- In Excel, we can build a table with various possible w's:

w	$E(\tilde{r}_P)$	σ_P
-0.5	2.50%	0.00%
0	10%	25%
0.5	17.50%	50.00%
1	25.00%	75.00%
1.5	32.50%	100.00%

Two Risky assets, $\rho_{AB} = 1$



- The picture looks very similar to the case where there are one risky and one riskless asset.
- Because the two assets are perfectly correlated, we can build a 'synthetic' riskless asset (but how is one of the weight to get the riskless asset ?).

Two Risky assets, $\rho_{AB} = -1$

- When $\rho_{AB} = -1$, we can again simplify the variance equation:

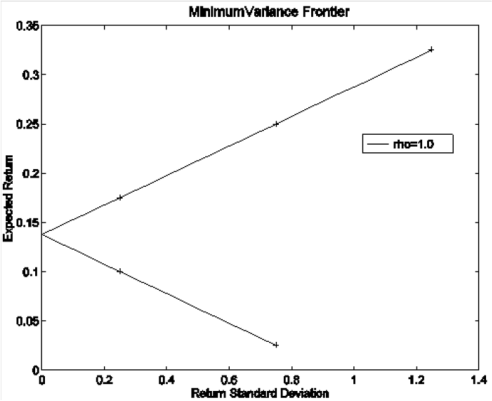
$$\sigma_P^2 = w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\rho_{AB}\sigma_A\sigma_B$$

$$\begin{aligned}\sigma_P^2 &= w^2\sigma_A^2 + (1-w)^2\sigma_B^2 - 2w(1-w)(\sigma_A\sigma_B) \\ &= (w\sigma_A - (1-w)\sigma_B)^2 \\ \sigma_P &= |w\sigma_A - (1-w)\sigma_B|\end{aligned}$$

- Again, if we create a table of the expected returns and variances for different weights and plot these, we get: (here for $0.5 \leq w \leq 1.5$):

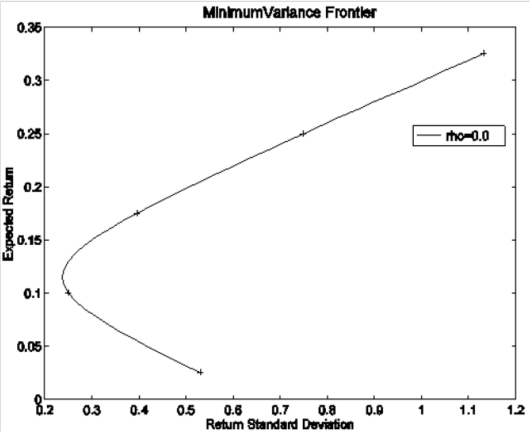
w	$E(\tilde{r}_P)$	σ_P
-0.5	2.5%	75.0%
0	10.0%	25.0%
0.25	13.75%	0.0%
0.5	17.5%	25.0%
1	25.0%	75.0%
1.5	32.5%	125.0%

Two Risky assets, $\rho_{AB} = -1$



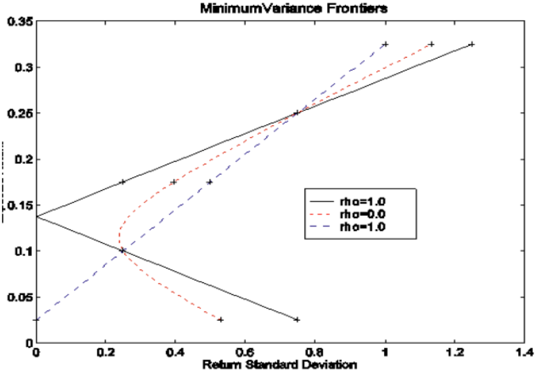
- Because the two assets are perfectly correlated, we can build a 'synthetic' riskless asset.
- Some combinations are 'dominated' in this case. Which ones?

Two Risky assets, $-1 < \rho_{AB} < +1$



- The plot shows that there is now some diversification effect (though not as much as when $\rho = 1$ or $\rho = -1$).

All cases together



- To calculate the minimum variance frontier in this 2 assets world you need the following: $E(\tilde{r}_A), E(\tilde{r}_B), \sigma_A, \sigma_B$ *and* ρ_{AB}

Two Risky Assets: Exercises

- You have invested \$20,000 in the SP 500, with an expected return of 15% and a risk of 20%. You are thinking of investing \$4,000 in gold, which has the same expected return and risk as the SP 500 but whose correlation with the SP 500 is equal to -1.

1. If you make this gold purchase, what will be the expected return and risk of your new portfolio?
   Hint: What is the weighting of gold in your portfolio? Use the formulas given for the expected return and risk of a portfolio.
2. What percentage must be invested in each of the two assets in order to obtain a zero-risk portfolio?
3. If the correlation between these two indices had been $\rho = +1$, what would be the risk of the initial portfolio?

- Download the file "2assetsstudents.xls" et answer the questions.

The CAL with Two Risky Assets

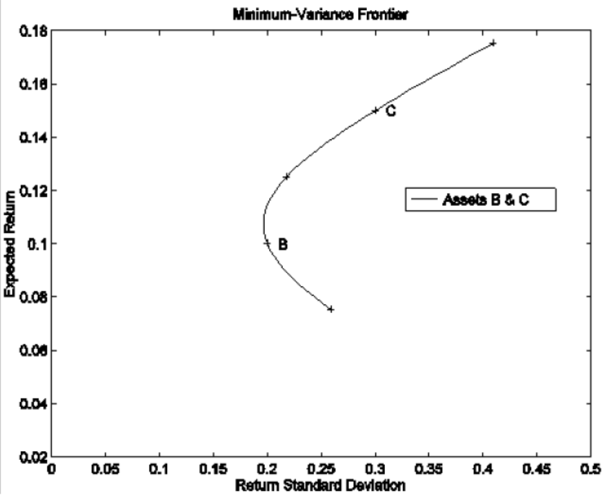
- For this section, let's assume we can only trade in the risk-free asset ($r_f = 3\%$) and risky assets B and C, where,

Asset	$E(\tilde{r})$	σ
B	10%	20%
C	15%	30%

and $\rho_{BC} = 0.5$

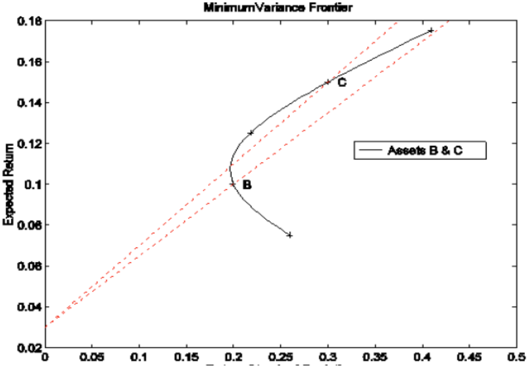
- We can compute the Minimum Variance frontier created by combinations of the B and C

The CAL with Two Risky Assets



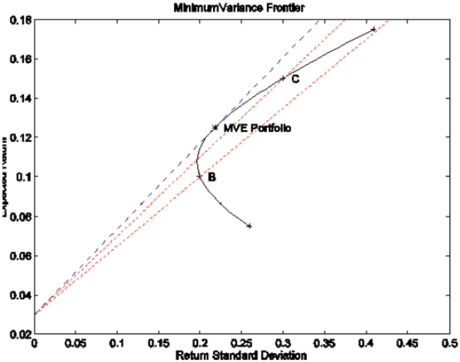
The CAL with Two Risky Assets

- Now, lets look at the situation where we can include either B, C or the risk-free asset in our portfolio.
- If we use either the risk-free asset plus asset B, or the risk-free asset plus asset C, the two possible CALs are:



The CAL with Two Risky Assets

- What is the optimal combination?



- We call this portfolio the tangency portfolio (why?). In combination with the risk-free asset,
  - it provides the "steepest" CAL (the one with the highest slope)
  - It is sometimes called the Mean-Variance Efficient or MVE portfolio.
  - Why is this the portfolio we want?

MVE portfolio

- How do we find the MVE portfolio mathematically?
- Find the portfolio with the highest Sharpe Ratio (Why?):

$$\max_{w} \frac{E\left(\tilde{r}_P\right) - r_f}{\sigma_P}$$

where

$$E\left(\tilde{r}_P\right) = wE\left(\tilde{r}_B\right) + (1 - w)\,E\left(\tilde{r}_C\right)$$

$$\sigma_P = \left[w^2\sigma_B^2 + (1 - w)^2\,\sigma_C^2 - 2w(1 - w)\sigma_B\sigma_C\rho_{BC}\right]^{1/2}$$

- Unfortunately, the solution is pretty complicated:

$$w_B^P = \frac{E\left(\tilde{r}_B^e\right)\sigma_C^2 - E\left(\tilde{r}_C^e\right) cov\left(\tilde{r}_B, \tilde{r}_C\right)}{E\left(\tilde{r}_B^e\right)\sigma_C^2 + E\left(\tilde{r}_C^e\right)\sigma_B^2 - \left[E\left(\tilde{r}_B^e\right) + E\left(\tilde{r}_C^e\right)\right] cov\left(\tilde{r}_B, \tilde{r}_C\right)}$$

MVE portfolio

- Here, $w_B^P = 0.5$, which gives a mean and a variance for this portfolio of $E(\tilde{r}_{MVE}) = 0.1250$ and $\sigma_{MVE} = 0.2179$.
- Also, the Sharpe Ratio of the MVE portfolio is:

$$SR_{MVE} = \frac{E(\tilde{r}^e_{MVE})}{\sigma_{MVE}} = \frac{0.095}{0.2179} = 0.4359$$

- Assets B and C have Sharpe Ratios of 0.35 and 0.40.
- The Sharpe Ratio of the MVE portfolio is higher than those of assets B and C. Will this always be the case?
- Have we determined the optimal allocation between risky and riskless assets?

MVF portfolio with many risky assets

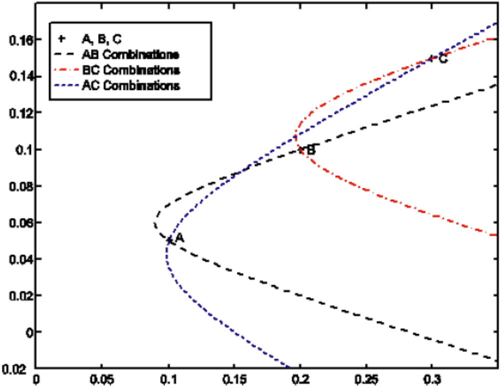
- Now let's look at the optimal portfolio problem when there are three assets.
- The expected returns, standard deviations, and correlation matrix are:

Asset	$E(\widetilde{r})$	σ
A	5%	10%
B	10%	20%
C	15%	30%

Correlations			
Assets	A	B	C
A	1	0	0.5
B	0	1	0.5
C	0.5	0.5	1

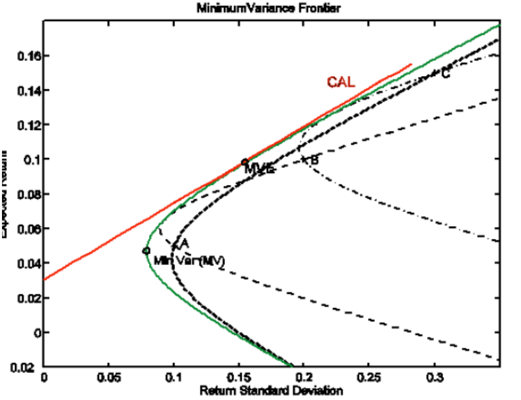
- What does the minimum variance frontier (MVF) look like now?

MVF portfolio with many risky assets



- If we combine A and B, B and C, or A and C, we get the above possible portfolio combinations.
- However, we can do better.

MVF portfolio with many risky assets



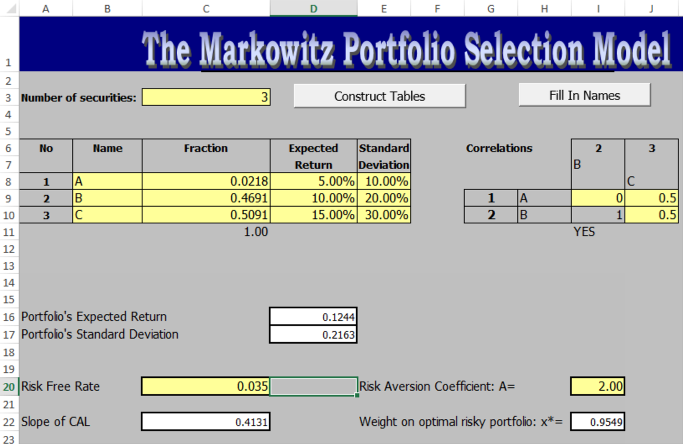
- This plot adds the mean-variance frontier and the CAL (red line) to the three-asset portfolios (green curve).
- This shows that the MVE portfolio will be a portfolio of portfolios, or a combination of all of the assets.

Allocating Capital among Multiple Risky Securities

- The mathematics of the problem quickly become complicated as we add more risky assets.
- We are going to need a general purpose method for solving the multiple asset problem.
- Fortunately Excel's "solver" function can solve the problem using the spreadsheet called Markowitz_Optimization.xlsm which can be found on the course homepage.

MVF portfolio with many risky assets

- Take the three risky assets A,B,C from before with $r_f = 3.5\%$.
- Just fill in the input data (yellow cells).The slope of the CAL, optimal weights w and $E(\tilde{r}_{MVE})$, σ_{MVE} can be calculated in Excel



The Markowitz Portfolio Selection Model

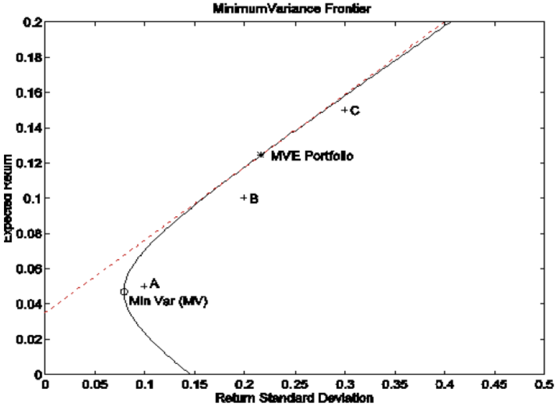
Number of securities: 3 | Construct Tables | Fill In Names

No	Name	Fraction	Expected Return	Standard Deviation
1	A	0.0218	5.00%	10.00%
2	B	0.4691	10.00%	20.00%
3	C	0.5091	15.00%	30.00%
		1.00		

Correlations		2 B	3 C
1	A	0	0.5
2	B	1	0.5
		YES	

Portfolio's Expected Return	0.1244
Portfolio's Standard Deviation	0.2163
Risk Free Rate	0.035
Risk Aversion Coefficient: A=	2.00
Slope of CAL	0.4131
Weight on optimal risky portfolio: x*=	0.9549

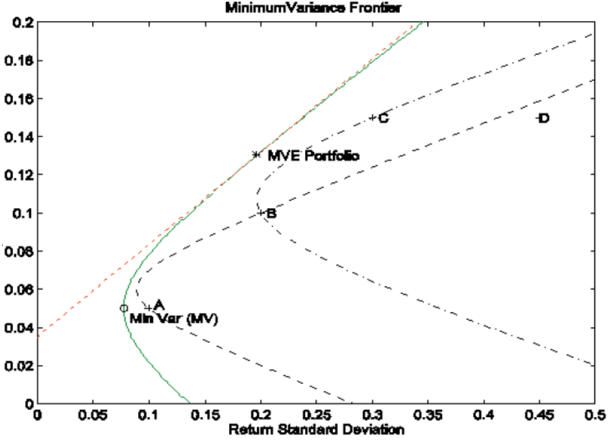
MVF portfolio with many risky assets



$$w_{MVE} = \begin{bmatrix} 0.0218 \\ 0.4691 \\ 0.5091 \end{bmatrix} \qquad \begin{aligned} E(r_{MVE}) &= 0.1244 \\ \sigma_{MVE} &= 0.2163 \\ SR_{MVE} &= 0.4131 \end{aligned}$$

MVF portfolio with many risky assets

- Let's add a fourth security, say, D with $E(\tilde{r}_D) = 15\%$, $\sigma_D = 45\%$.
- Assume that it is a zero correlation with all of the other securities.
- Will anyone want to hold this security?



MVF portfolio with many risky assets

- Excel ouput looks like:

The Markowitz Portfolio Selection Model

Number of securities: 4

Construct Tables | Fill In Names

No	Name	Fraction	Expected Return	Standard Deviation
1	A	0.0168	5.00%	10.00%
2	B	0.3616	10.00%	20.00%
3	C	0.3924	15.00%	30.00%
4	D	0.2292	15.00%	45.00%
		1.00		

Correlations		2 B	3 C	4 D
1	A	0	0.5	0
2	B	1	0.5	0
3	C		1	0
		YES		

Portfolio's Expected Return: 0.1302

Portfolio's Standard Deviation: 0.1961

Risk Free Rate: 0.035

Risk Aversion Coefficient: A= 2.00

Slope of CAL: 0.4858

Weight on optimal risky portfolio: x*= 1.2388

MVF portfolio with many risky assets

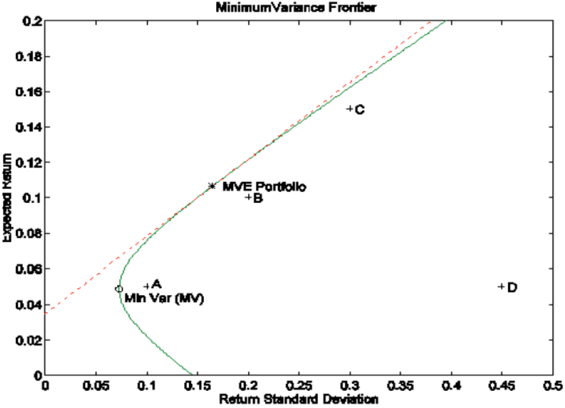
- The optimal allocation looks like this:

$$w_{MVE} = \begin{bmatrix} 0.0168 \\ 0.3616 \\ 0.3924 \\ 0.2292 \end{bmatrix} \qquad \begin{aligned} E(\tilde{r}_{MVE}) &= 0.1302 \\ \sigma_{MVE} &= 0.1961 \\ SR_{MVE} &= 0.4858 \end{aligned}$$

- What explains this?
  - D is apparently strictly dominated by C
  - D is uncorrelated with A, B, C
  - How can D contribute to the overall portfolio?
  - Wouldn't increasing the share of C by 22.92% dominate the allocation above (check it on excel sheet)?

MVF portfolio with many risky assets

- Let's change the fourth security. Suppose D has $E(\tilde{r}_D) = 5\%$, $\sigma_D = 45\%$
- Assume that it is a correlation of -0.2 with all of the other securities.
- Will anyone want to hold D now (Do it on excel)?



MVF portfolio with many risky assets

- The new optimal allocation looks like this

$$w_{MVE} = \begin{bmatrix} 0.1215 \\ 0.3925 \\ 0.3685 \\ 0.1175 \end{bmatrix} \qquad \begin{aligned} E(\tilde{r}_{MVE}) &= 0.1065 \\ \sigma_{MVE} &= 0.1646 \\ SR_{MVE} &= 0.4342 \end{aligned}$$

- Compare to previous ($E(\tilde{r}_D) = 15\%$, zero correlation):

$$w_{MVE} = \begin{bmatrix} 0.0168 \\ 0.3616 \\ 0.3924 \\ 0.2292 \end{bmatrix} \qquad \begin{aligned} E(\tilde{r}_{MVE}) &= 0.1302 \\ \sigma_{MVE} &= 0.1961 \\ SR_{MVE} &= 0.4858 \end{aligned}$$

- Why are we still holding D?
- Are we worse off with the new D (compare to the intial situation without D)?
- Basic Message: Your risk/return tradeoff is improved by holding many assets with less than perfect correlation.

Understanding Diversification

1. Start with our equation for variance:

$$\sigma_P^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = \sum_{i=1}^{n-1}\sum_{\substack{j=i \\ j\neq i}}^{n} w_i w_j \sigma_{ij} + \sum_{i=1}^{n} w_i^2 \sigma_i^2$$

2. Then make the simplifying assumption that $w_i = 1/n$ for all assets

$$\sigma_P^2 = \sum_{i=1}^{n}\left(\frac{1}{n}\right)^2 \sigma_i^2 + \sum_{i=1}^{n-1}\left(\frac{1}{n}\right)^2 \sum_{\substack{j=1 \\ j\neq i}}^{n} \sigma_{ij}$$

3. The average variance and covariance of the securities are:

$$\overline{\sigma^2} = \left(\frac{1}{n}\right)\sum_{i=1}^{n}\sigma^2 \text{ and } \overline{cov} = \left(\frac{1}{n}\right)\sum_{\substack{j=1 \\ j\neq i}}^{n}\sigma_{ij}$$

4. Plugging these into our equation gives:

$$\sigma_P^2 = \left(\frac{1}{n}\right)\overline{\sigma}_i^2 + \frac{(n-1)}{n}\overline{cov}$$

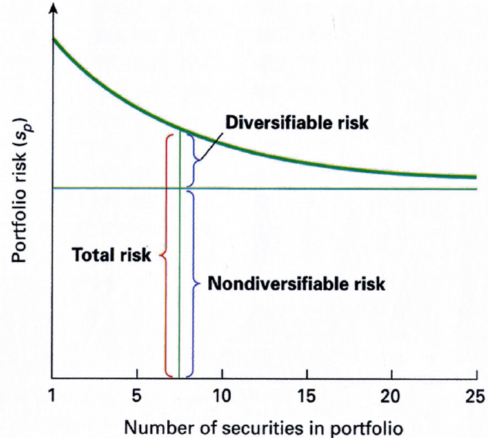
5. What happens as N becomes large?

$$\frac{1}{n} \rightarrow 0 \text{ and } \frac{n-1}{n} \rightarrow 1$$

6. Only the average covariance matters for large portfolios.
7. If the average covariance is zero, then the portfolio variance is close to zero for large portfolios

Understanding Diversification

- This plot shows how the average standard deviation of a portfolio of stocks changes as we change the number of assets in the portfolio.



Understanding Diversification

- The component of risk that can be diversified away is the diversifiable or non-systematic risk.
- Empirical Facts
  - The average (annual) stock return standard deviation is 49%
  - The average (annual) covariance between stocks is 0.037, and the average correlation is about 39%.
- Since the average covariance is positive, even a very large portfolio of stocks will be risky. We call the risk that cannot be diversified away the systematic risk.

Conclusion

In this lecture we have developed mean-variance portfolio analysis.
1. We call it mean-variance analysis because we assume that all that matters to investors is the average return and the return variance of their portfolio.

- This is appropriate if returns are normally distributed.

2. There are a couple of key lessons from mean-variance analysis:

- You should hold the same portfolio of risky assets no matter what your tolerance for risk.
  * If you want less risk, combine this portfolio with investment in the risk-free asset.
  * I If you want more risk, buy the portfolio on margin.
- In large portfolios, covariance is important, not variance.

Conclusion

What is wrong with mean-variance analysis?

- Not much! This is one of the few things in finance about which there is complete agreement.

- Caveat: remember that you have to include every asset you have in the analysis; including human capital, real estate, etc.

- Finally, Markowitz's theory tells us nothing about where the prices, returns, variances or covariances come from.

  - This is what we will spend much of the rest of the course on!

- In the next lecture we will investigate Equilibrium Theory
- Equilibrium theory takes Markowitz portfolio theory, and extends it to determine how prices must be set in an efficient market.